NOTES FOR GROUNDWATER SECTION OFFICERS

By

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Groundwater Hydrology

In the Groundwater Section of the Water Resources Branch over a period of time certain of the groundwater evaluation methods have proved of more value. When working with the associated mathematical formulae a convenient set of units closely related to the field units commonly used in the Territory have been adopted. This paper sets out the units used and states the most frequently used formulae together with some comment on their application.

Units

The use of what is known as a consistent "set of units" has many advantages especially for the mathematician engaged on theoretical evaluation of groundwater flow or for the hydrologist desiring to compare his results with overseas experience but for most officers of the Branch the calculation to convert field measurements to the consistent units is more tedious than the advantage justifies. Therefore the units adopted for day to day work in the groundwater section are very close to the normal units used in the Territory.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit of Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>Drawdown</td>
<td>Feet</td>
</tr>
<tr>
<td>T</td>
<td>Transmissibility equals Km</td>
<td>Thousands of imperial gallons per day per foot of aquifer width.</td>
</tr>
<tr>
<td>X</td>
<td>Permeability</td>
<td>Thousands of imperial gallons per day per square foot of aquifer cross section</td>
</tr>
<tr>
<td>m</td>
<td>Aquifer thickness</td>
<td>Feet</td>
</tr>
<tr>
<td>S</td>
<td>Storage coefficient</td>
<td>(dimensionless)</td>
</tr>
<tr>
<td>Q</td>
<td>Pumping rate or flow rate</td>
<td>Thousands of imperial gallons per hour</td>
</tr>
</tbody>
</table>
### Symbol Description Unit of Measurement

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<th>Symbol</th>
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<tbody>
<tr>
<td>r</td>
<td>Radius</td>
<td>Feet</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Effective radius of the well (or bore)</td>
<td>Feet</td>
</tr>
<tr>
<td>$r_e$</td>
<td>Radius of extreme limit of cone of depression</td>
<td>Feet</td>
</tr>
<tr>
<td>t</td>
<td>Time (usually from commencement of pumping)</td>
<td>Minutes</td>
</tr>
<tr>
<td>$t_1$</td>
<td>Time from cessation of pumping</td>
<td>Minutes</td>
</tr>
<tr>
<td>$t_o$</td>
<td>Time at which drawdown stabilizes</td>
<td>Minutes</td>
</tr>
<tr>
<td>$t_o$</td>
<td>Value on time scale at which the straight line portion of a semi logarithmic drawdown plot does (if produced back) intersect the zero drawdown axis</td>
<td>Minutes</td>
</tr>
<tr>
<td>$Z$</td>
<td>Slope of the straight line portion of a semi logarithmic time drawdown cycle plot</td>
<td>Feet per log of time</td>
</tr>
<tr>
<td>$Z_r$</td>
<td>Slope of the semi-logarithmic radius of drawdown plot</td>
<td>Feet per log-cycle</td>
</tr>
<tr>
<td>$i$</td>
<td>Gradient (of piezometric surface)</td>
<td>(dimensionless)</td>
</tr>
<tr>
<td>F</td>
<td>Extraction frontage of a well</td>
<td>Feet</td>
</tr>
<tr>
<td>L</td>
<td>Width of flow path or width of aquifer</td>
<td>Feet</td>
</tr>
<tr>
<td>a</td>
<td>Distance from pumped well to a boundary</td>
<td>Feet</td>
</tr>
<tr>
<td>u</td>
<td>Well characteristic</td>
<td></td>
</tr>
</tbody>
</table>

\[
u = \frac{2.25 \times r^2 \times s}{t_o} = \frac{t_o}{1.755}
\]

### Symbols as defined in the expression

\[s = Bq + Cq^2\]

which expresses drawdown as made up of two components, one a head loss.
3.

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<td></td>
<td>due to laminar flow in the aquifer and linear with ( q ), the other a head loss due to turbulent flow in the well casing, entry zone or the aquifer, proportional to ( q^2 )</td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td>Varies with time and after appreciable time has elapsed may be expressed approximately as:</td>
<td>Feet per thousand imperial gallons per hour</td>
</tr>
<tr>
<td></td>
<td>( B = \frac{4.4}{T} \log \frac{t}{t_0} )</td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>Well-loss constant</td>
<td>Feet per (thousands of imperial gallons per hour) squared</td>
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**Pump Test Analysis and Application**

The initial step in any groundwater appraisal is the determination of the aquifer characteristics, Transmissibility (\( T \)) and Storage Coefficient (\( S \)).

The most simple and reliable method of determining these is by analysis of pump test results.

To ensure that the readings being taken in the field are forming an intelligent pattern and for most hydrologic analyses, the simple approximate plot known as the Jacob Approximation is used. This method requires the plotting of a graph on semi-logarithmic paper plotting the drawdown on the linear scale (with the zero drawdown axis along the top edge of the paper) and the time since pumping commenced along the logarithmic scale (Figure 1).

This method has inaccuracies and the early readings cannot be expected to fall on a straight line. After this initial period usually it will be possible to draw a straight line through the plotted points. To carry out computations based on this plot it is necessary to measure the slope (the increase in drawdown for each log-cycle of time, i.e. from 10 to 100 or from 100 to 1,000 minutes) and to draw in the straight line projecting it back to give the intercept with the zero drawdown axis (this intercept gives the whole value of \( t_0 \)). The approximation is valid for values of
"u" less than 0.01 or, in other words provided that the value of 'u' of the point plotted is more than \( \frac{1}{3} \) of a log cycle greater than the value of 't' revealed by the plot.

From the slope of the straight line (2) the Transmissibility may be determined,

\[
T = \frac{4.442}{2}
\]  

(1)

and from the zero drawdown intercept if the readings were taken in an observation hole at radius 'r' feet the Storage Coefficient can be calculated.

\[
s = \frac{r}{4x} \log \frac{t}{t_0}
\]  

(2)

A rough approximation of the storage coefficient can be achieved by using the results plotted from measurements in the pumped bore provided that the well loss is allowed for and if it can be assumed that the effective radius of the bore equals the drilled radius.

The equation describing this straight line is:

\[
s = \frac{4.442}{2} \log \frac{t}{t_0}
\]  

(3)

OR

\[
s = \frac{4.442}{2} \log \frac{t}{t_0}
\]  

(4)

OR

\[
s = \frac{4.442}{2} \log \frac{1}{1.78u}
\]  

(5)

OR

\[
s = BQ
\]  

(6)

Equation (4) is useful for determining the drawdown at any time for any pumping rate. Equation (3) being more general can be used for determining the expected drawdown at any point within the influence of the pumped bore at any time or pumping rate. (These formulae must be used with caution in a pumped hole because they make no allowance for well losses)

One advantage of the straight line plot is the clarity with which it reveals boundaries. In figure (1) an impermeable boundary effect can be seen at 80 minutes. At this point the slope of the plot exactly doubles
since the effect of the boundary can be regarded as equivalent to the effect of an imaginary second well pumping at the same rate as the real well.

The distance to the boundary can be calculated. If \( t \) is the time at which the boundary effect occurs then the distance from the pumped well is given by

\[
\text{Distance to boundary} = \sqrt{\frac{2 \times \text{const.}}{t}}
\]

where \( r \) and \( t_0 \) are the figures for the observation hole which has been plotted.

Frequently the drawdown plot will "level out" either to become a line of constant drawdown or to continue to show an increasing drawdown but at a reduced rate such as indicated by the dashed line in Figure 1. This curve which is referred to as having a flattened 'Z' shape can result from one or more of many situations, among these being:

(a) Effects of partial penetration;
(b) Leakage through or from a semipervious layer above or below the aquifer;
(c) Effects of lateral variation in aquifer permeability;
(d) Interception of a recharge boundary;
(e) Interception of underflow through an aquifer having a sloping piezometric surface;
(f) Effects of aquifer shape (non-uniform thickness for example).

Thus it is essential that the nature of the aquifer be known if an accurate analysis of the test is to be achieved. Some assistance may be derived by recognizing that the point of slope flattening will occur at a constant time irrespective of pumping rate if the effect results from the geometry of the aquifer but the time of flattening will vary with pumping rate if the flattening is caused by interception of underflow or leakage through the aquifer confining beds.

In the case of interception of underflow the drawdown will be constant once sufficient underflow is intercepted and the transmissibility of the formation should be calculated from the slope of the plot immediately prior to the flattening.
In the case of partial penetration or leakage from another confined bed (which are very similar situations) the transmissibility of the whole aquifer series is calculated from the slope of the plot after the flattening. In fact where there are problems of aquifer geometry on completion of the bore to expose only portion of a complex aquifer series most reliable results are obtained by ignoring the early readings and computing from readings plotted during the period after (say) one week with the test extending to ten weeks. These readings will give good figures for transmissibility but cannot be used to compute the storage coefficient unless the observation bore is sufficiently far from the pumped hole to escape any partial penetration effects. A minimum distance of 1/2 times the total aquifer thickness has been suggested. All bores closer than this (including the pumped hole) can be expected to show a drawdown plot having an increased drawdown (compared to the drawdown in a bore fully penetrating the aquifer) due to the partial penetration losses. Any attempt to calculate a Storage Coefficient from these curves (unless the superimposed losses can be determined with accuracy and an adjustment made) must give a low result.

The effective radius of the cone of depression at the time of flattening can be calculated from:

$$r_e^2 = \frac{r_0^2 t_e}{t_o}$$  \hspace{1cm} (8)

If the flattening is due to constant drawdown and it has been established that this results from interception of underflow then, approximately, the gradient on the piezometric surface is given by:

$$1 = \frac{3.82Q}{2\pi r_e}$$  \hspace{1cm} (9)

The extraction frontage of the well can also be calculated:

$$F = 6.28 r_e$$  \hspace{1cm} (10)

Using the information obtained from equations (1) and (8), if an aquifer of limited width (L) can be recognised then the total flow through the aquifer is given by:

$$\text{Total Flow} = \frac{\pi L L}{24}$$  \hspace{1cm} (11)
More accurate results, particularly when leakage is involved or when only very short term tests are available, will be given if the readings are plotted on log-log paper plotting drawdown, vertically and time along the horizontal axis. (Thesis Method)

This analysis depends on the use of a "type curve" which has been prepared from the more precise but complex integral expression involving the well characteristic "u" and an exponential integral function known as the "Well Function of u" and written "W(u)".

In the units used in the Northern Territory the complex expression reduces to:

\[ s = \frac{1.210}{2} \cdot W(u) \quad \text{OR} \quad \log s = \log \frac{1.210}{2} + \log W(u) \quad (12) \]

and

\[ \frac{s}{r^2} = \frac{2.258}{2} \cdot \frac{1}{u} \quad \text{OR} \quad \log \frac{s}{r^2} = \log \frac{2.258}{2} + \log \frac{1}{u} \quad (13) \]

When the experimental plot of \( s \) against \( t \) is fitted to the type curve plot of \( W(u) \) against \( \frac{1}{u} \) (all in logarithmic scales) the axis of the graphs will be displaced in a vertical direction by the amount \( \frac{1.210}{2} \) and in the horizontal direction by \( \frac{2.258}{2} \).

By selecting a convenient match point anywhere on the overlapping sheets and substituting the co-ordinates of this point in equations 11 and 12 above, both \( T \) and \( S \) are determined.

The same plot used with a special type curve can be used to determine leakage characteristics.

If the drawdown has achieved stability, or if sufficient time has elapsed for the shape of the cone to become stable and all readings are taken at the same time, then the Equilibrium Formula often provides a simple method of establishing \( T \).

Drawdown is measured at two observation points, drawdown \( s_1 \) at a bore at radius \( r_1 \) and drawdown \( s_2 \) at a bore at radius \( r_2 \). Then:

\[ T = \frac{8.96 \log \frac{r_1}{s_1 - s_2}}{s_1 - s_2} \quad (14) \]

The readings taken in several bores at the same time (or after equilibrium conditions have become established can be plotted on semi-logarithmic paper with the drawdown on the linear scale and the radius
to the observation bore on the logarithmic scale. A straight line plot will result. If $Z_T$ is the slope (feet of drawdown per log cycle of radius) of this straight line then:

$$T = \frac{S_B}{Z_T}$$  \hfill (15)

**Constant Drawdown Test**

A bore pumped at the fork (on a flowing artesian bore) produces water with the aquifer subjected to a constant head (provided that the "well-losses" are so small as to be negligible). As the cone increases in size the yield of the bore will decline in keeping with the approximate relationship (after an initial time) -

$$Q = \frac{S_B}{4.4 \log \frac{4r^2}{w^2}}$$  \hfill (16)

If the constant drawdown ($s$) is divided by the flow rate at any time $t$ then the relationship is -

$$\frac{s}{Q} = \frac{4.4}{S_B} \log \frac{4t}{4r^2 S}$$  \hfill (17)

Plotting the function $\frac{s}{Q}$ on the linear scale of a semilogarithmic paper and time on the logarithmic scale, a straight line plot will be obtained. If the slope per log cycle is $Z_q$ (feet per thousand gallons per hour) then

$$T = \frac{4.4}{Z_q}$$  \hfill (18)

The function of $\frac{s}{Q}$ is known as specific drawdown and the specific drawdown against time plot can be a useful tool in many test analyses. The value of $t_{50}$ given by this method is identical to that obtained from the more usual "s" against "t" plotting and equation (2) will give the Storage Coefficient, provided that the effective radius of the well is available.

It is useful to note that

$$\frac{s}{Z_q} = \log \frac{t}{t_0}$$  \hfill (19)
The analysis of a constant drawdown test by the above method is limited to bores having a negligible well-loss. Although the value of the well-loss usually is determined by a series of tests at different (but constant) rates it is possible to analyse a constant drawdown test to provide a figure for the well loss constant 'C'. This method is of reasonable accuracy if the flow rate has undergone a large variation during the test but provides little more than an indication of the order of value of 'C' when the range of variation is small.

The drawdown in a pumped bore is made up of two components, the genuine formation head loss and the friction or turbulent head loss. The usual equation for this relationship \( s = BQ + CQ^2 \) is discussed under the step-drawdown test (Equation 21).

Adopting the approximate (Jacob and Lohmar) relationship

\[
B = \frac{B_0 + B_1 \log \frac{t}{t_0}}{2}
\]

we see that 'B' will increase with time, as will \( \frac{B_0}{Q} \) provided that B has a value greater than 1, that is when \( \log \frac{t}{t_0} \) is greater than \( \frac{1}{4.4} \). Normally a \( \frac{B_0}{Q} \) against log t plot will be safely within this range but due to the variation in the factor \( CQ^2 \) as time increases the points plotted will lie on a curve convex toward the zero \( \frac{B_0}{Q} \) axis.

Having plotted the points, construct a 'fair' curve through the points rejecting points obviously not conforming to the general trend.

Select three points lying on this 'fair' curve, either one log cycle apart or any convenient fraction (e.g. one half) of a log cycle apart.

Then if the first point is at time \( t_1 \)

\[
\frac{B_0}{Q} = B_1 + CQ_1
\]

At time \( t_2 \) which equals \( t_1 + (1 \text{ log cycle}) \) i.e. \( 10t_1 \)

\[
\frac{B_0}{Q} = (B_1 + Z_1) + CQ_2
\]

where \( Z_1 \) is the slope of the theoretical \( \frac{1}{Q} \) against 't' plot (refer equation 17)

And at time \( t_3 \) which equals \( t_1 + (2 \text{ log cycles}) \) i.e. \( 100t_1 \)
From these three equations a solution is obtained to give figures for

'Zq', which equals \( \frac{4\pi}{f} \)

and for 'C'.

The calculations can be extended to obtain 't_o' and thence, if a
figure for 'xw' is available, the storage coefficient

Recovery Formula

The non-equilibrium (straight line approximation) formula is applicable
to the recovery of a pumped well which prior to shut-down was producing at
a constant rate 'Q'. If 's' is the residual drawdown (i.e. the drawdown
below S.W.L. not the recovery above the pumping level) then

\[
s = \frac{4\pi}{T} \log \frac{t}{t_o}
\]  

(20)

It will be recognized from this that theoretically a well will only

fully recover in infinite time i.e. when \( \frac{t}{t_o} = 1 \)

If values of 's' are plotted on the linear scale of semilogarithmic
paper and \( \frac{t}{t_o} \) on the log scale the straight line plot will have the slope

'\frac{Z}{T}' feet per log cycle of \( \frac{t}{t_o} \) and the straight line, in theory will cross
the zero axis at \( \frac{t}{t_o} = 1 \).

Transmissibility as before, is given by:

\[
T = \frac{4\pi}{S}
\]

but the calculation of 'S' using this method is not so simple. If a determina-
tion of 'S' is desired it is necessary to know the drawdown at the precise moment
of cessation of pumping. If this line is drawn into the plot the straight
line of the 's' against \( \log \frac{t}{t_o} \) plot can be produced downwards to give an inter-
cept at \( \frac{t}{t_o} \) value \( \frac{t}{t_o} = \circ \) where the line cuts the value of drawdown at
cessation of pumping. It is also necessary to know 't_o', the time from
commencement to cessation of pumping. Then,

\[
S = \frac{T t_o}{4\pi} \frac{S}{t_o}
\]  

(21)
11.

This formula is of no use if the drawdown at cessation of pumping contained any well loss component.

It should be noted that the recovery of a bore intercepting underflow commences with the cone having the same dimensions as existed at the time of interception of underflow. Therefore the cessation of pumping should be regarded as taking place at the time of interception and \( t \) and \( t' \) accordingly.

**Step Drawdown Tests**

These tests are capable of precise analysis only when a long term test has proved that no boundary conditions will affect the drawdown during the duration of the step testing. Then boundary conditions exist a series of separate rate tests is required.

The single rate tests described earlier are restricted in their application to the calculation of drawdown in a pumped well because they ignore the well loss which exists in all pumped wells. Then the equipment for a bore is being designed it is essential that the pumping depths at all desired yields should be available.

The drawdown in a pumped bore will conform approximately to the relationship

\[
s = 3Q + CQ^2
\]  

(22)

This expression suggests that the drawdown is made up of two components, one a head loss proportional to \( Q \) which is attributed to losses resulting from the linear flow of water through the aquifer and a second, resulting from turbulent losses in the casing, the perforations or screen as well as that part of the aquifer close to the bore where flow velocities are high, proportional to \( Q^2 \).

In some aquifers such as limestones, much of the aquifer loss is due to flow of a turbulent nature and then it has been suggested that a better expression is:

\[
s = CQ^n
\]  

(23)

where \( 'n' \) is a power of \( Q \) greater than 1.

Unless \( 'n' \) is found to be very close to 2 (say greater than 1.35)
this expression is risky as it is certain that if the pumping rate is increased some turbulence must occur.

The relationship between 's' and 'q' is determined by a "step" or "multiple-stage" pumping test in which readings of drawdown are taken while the bore is pumped for successive periods at three or four rates with the change of rate being made rapidly at the end of each period. These tests are designed to avoid the expense of four separate different rate tests with the necessary recovery period between each test, but if any boundary condition exists or if the prior long single rate test had revealed a flattened Z characteristic than separate tests are essential. The exception to this is the case where the flattened Z characteristic is caused by the interception of underflow, here a step test is satisfactory provided each step goes to stability.

C.E. Jacob who designed the step-drawdown test suggested that each increment of drawdown be associated with the increment of flow-rate, causing it, and analysed the early stages of each step as a superimposition on the preceding step. This requires a pump unit capable of widely differing pumping rates.

The pumping equipment available in Australia is better suited to an analysis permitting computation using the total pumping rate. By recognizing that after an increment of pumping rate is imposed the drawdown rapidly becomes asymptotic to the drawdown which would have applied if the bore had been pumped at the new rate since pumping commenced, provided that steps of sufficient duration are adopted the correct total drawdown for each total pumping rate can be read from a semi-logarithmic drawdown plot.

The aqifier loss component of drawdown is variable, increasing with time, therefore for the purpose of this analysis it is essential that the readings to be used in the computation be read at the same time line on the plot. This makes B a constant. Then from equation (21)

$$\frac{\Delta s}{q} = B + Cq$$  \hspace{1cm} (24)

If from the readings of 's'! and 'q'! extracted from the semilogarithmic plot $\frac{\Delta s}{q}$ is calculated and then $\frac{\Delta s}{q}$ is plotted on rectilinear graph paper against 'q', a straight line graph will result with the line having a slope
(units of \( \frac{q}{s} \) per 1,000 g.p.h.) equal to '0' and cutting the zero Q axis at a value of \( \frac{S}{Q} \) equal to 'B' at the time the readings were taken.

If the \( \frac{S}{Q} \) against Q plot does not give a truly straight line but is inclined to be a curve convex upward (towards higher values of \( \frac{S}{Q} \)) then it is possible that there is little or no truly linear flow. Flow through joints or solution cavities is more accurately expressed by equation (22).

To check this, plot 's' on one axis against Q on the other axis on log-log paper. A straight line, on this plot gives the value of 'C' at the intercept of the graph with the \( Q = 1 \) line and 'n' is given by the slope of the graph. A log-log plot which is concave upward (towards higher values of 's') suggests that the relationship of equation (21) is correct.

The effective radius of a pumped well can be calculated using the value of 'B' obtained from the \( \frac{S}{Q} \) against Q plot (assuming that the value for S is available from readings in an observation hole) by substituting figures to T, S and t (the time at which the drawdown readings were taken) in the expression

\[
B = \frac{4.4}{T} \log \frac{Tt}{4\pi S}
\]  

(35)

Determination of the effective radius is a positive check on the efficiency of development of a bore. It has been suggested that the value of 'C' itself is an adequate check. W.C. Walton (Illinois State Water Survey) suggests that a properly developed well will have a value of 'C' (in our units) of less than 0.01, C values between 0.01 and 0.02 being indicative of mild deterioration and severe clogging being present if \( C \) is greater than 0.02. A value of greater than 0.08 would indicate irreparable damage.

Northern Territory experience where many bores produce from joints rather than interstitial permeability is that values of 'C' up to about 0.3 are acceptable. Determination of 'C' at intervals during development will permit a check on whether the work is improving the performance of the well and this indication is of more use than the absolute value of 'C' as a criterion of development.
Water Level (or Piezometric Surface Variations)

Since the drawdown at a pumped well or observation hole is the reduction in water level due to pumping it is essential that any variations in water level during the course of the test be recognized and allowed for. A long term trend of variation can be examined before and after the test and the intermediate values estimated. It is of value however if an observation bore within the aquifer but outside the zone of influence of the pumped bore can be measured during the test.

Water Table Aquifer

Most testing in the Northern Territory involves pumping from confined or at least partially confined aquifers. Thus variations in the piezometric surface close to the bore within the zone of depression do not affect the transmissibility of the aquifer.

When a true water table aquifer is pumped the water level reduction within the zone of depression will cause a reduced transmissibility at this point and this will result in increased drawdowns. The transmissibility computed for the aquifer in consequence will be below the correct figure unless allowance is made for the water level reduction, particularly when the drawdown is large compared to the initial depth of saturated aquifer.

Jacob suggested a means of obtaining a reasonable figure for 'T' even when dewatering is as much as 25% of the original thickness. The observed drawdown is reduced by the quantity

\[ \frac{m^2}{2m} \]

where \( m \) is the observed drawdown and \( m \) is the initial depth of saturated aquifer.

If adjusted values of 's' are plotted on the \( s-\log \) plot reasonable values of both T and S can be computed, using equations (1) and (2).
NOTE THAT 10^6 MINUTES IS APPROX. 1 WEEK AND CONSEQUENTLY LONG TERM TESTS ARE CONVENIENTLY PLOTTED IN WEEKS.

5-4/17